3943

31 March 1969

Materiel Test Procedure 5-1-031
White Sands Missile Range

U. S. ARMY TEST AND EVALUATION COMMAND
BACKGROUND DOCUMENT

CINETHEODOLITES

1. INTRODUCTION

One of the objectives of testing missile and rocket systems is to determine the actual "in-flight" performance of the vehicles themselves. One of the prime requirements for establishing the performance of vehicles in flight is to obtain accurate data which will reveal the position in space and the attitude of the vehicle during its trajectory. The employment of optics at a missile range may become highly significant in obtaining these data if the atmosphere permits reasonably unobstructed observation; and if, moreover, an all-land test area makes possible optimum siting of instruments for most desirable look angles. Under these conditions, optics in general, and photogrammetry in particular, correlated with other instrumentation systems, can provide effective and accurate data of target trajectory. Table I offers data of the optical trajectory systems capabilities at a missile range.

Cinetheodolites are probably the most important and widely used class of photographic instruments employed for the collection of trajectory data, due to their versatility and their coverage and accuracy capabilities. These instruments were developed from a family of optical devices known as theodolites by the addition of recording motion picture type cameras which can be either cine or pulse operated; and which give cinetheodolites the ability to track the vehicle in flight and to obtain continuous trajectory data. Figure 1 illustrates a typical permanent cinetheodolite station with astrodome. Figure 2 shows a mobile cinetheodolite mount with instrument installed. Table II provides data on characteristics of cinetheodolites in actual use at a missile range.

Cinetheodolites provide angular measurements of the line of sight to the vehicle. This permits acquiring accurate position data. Together with timing systems, velocity and acceleration data can be developed from the position measurements. Characteristics of cinetheodolites make it possible for these instruments to serve as primary sources of position and velocity data from 2000 feet (615 meters) slant range to approximately 100,000 feet (30,769 meters) slant range. Cinetheodolites can also provide secondary coverage between lift-off and 2000 feet slant range.

Cinetheodolites can be used to acquire data in the testing of missiles, rockets, projectiles, aircraft, and fire control systems; in the ripple firing of rockets, graze action tests, airburst fuze tests, and similar operations. During these activities, cinetheodolites can provide photographic records of the vehicle or target image, the azimuth and elevation readings of the cinetheodolite dials, the film frame count, and the binary real time of film exposures. Figure 3 illustrates a frame of a cinetheodolite film.

Table I. Trajectory Systems Capabilities
(Optical)

System	Trajectory Phase	Sample Rate (samples/sec)	Position (ft)	Velocity (ft/sec)	Acceleration (ft/sec^2)
Fixed Camera (Ribbon Frame)	Launch	up to 180	0.5-1.0	4.0	8.0
	Boost	up to 180	2.5-5.0	6.0	16.0
	Terminal (0 to 1000 ft. only)	up to 180	1.0	4.0	8.0
Fixed Camera 70mm Pin Registered Cameras	Launch	up to 80	0.5-1.0	4.0	8.0
	Boost	up to 80	2.5	6.0	16.0
	Terminal	up to 80	1.0	4.0	8.0
Fixed Camera 70mm Rotating prism Cameras	Launch	up to 360	0.5-1.0	4.0	8.0
	Boost	up to 360	2.5	6.0	16.0
	Terminal	up to 360	1.0	4.0	8.0
Fixed Camera 35mm Rotating prism Cameras	Launch	up to 2500	0.5-1.0	4.0	8.0
	Boost	up to 2500	2.5	6.0	16.0
Cinetheodolite (Contraves)	Boost	5/10/20/30	1-3	2-5.0	10.0
	Mid-course	5/10/20/30	2-9	4-8	15.0
Cinetheodolite (Askania)	Boost	up to 5	1-10	5.0-10	10.0-15
	Mid-course	up to 5	5-15	10-15.0	15-20.0
	Terminal	up to 5	5-15	10-15.0	15-20.0
Cinetheodolites (BRL/NGF)	Mid-course (60,000 Ft. max.)	5/10/20/40	10-20	15-25	20-30
		5/10/20/40	10-20	15-25	20-30
Ballistic Cameras	Mid-course	2-20 (or dependent on beacon)	1-5	5.0*	5.0*

* Not normally provided; however, when required for special purpose these data can be obtained.

Figure 1. Typical Cinetheodolite Station with Astrodome.

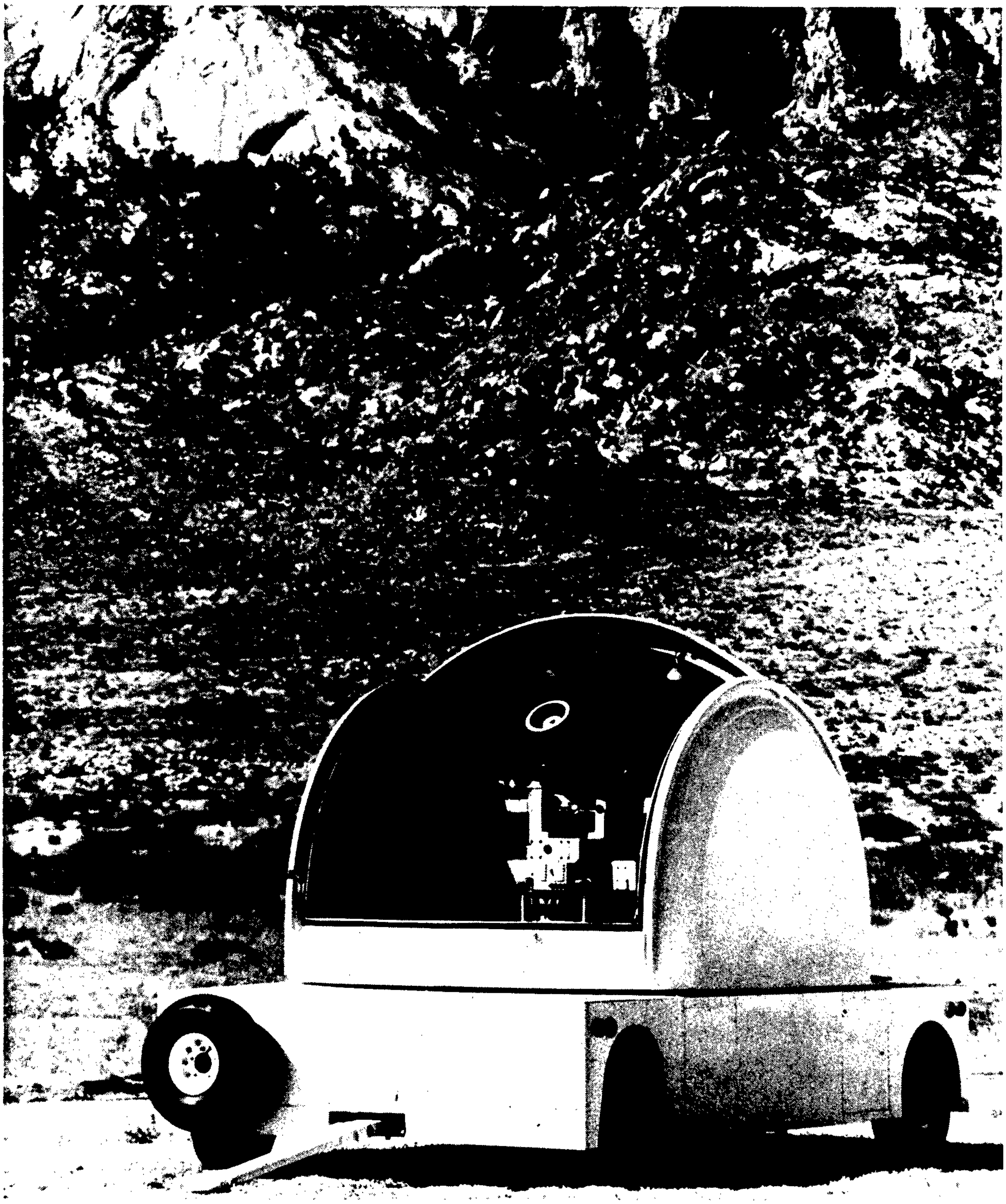

Figure 2. A Mobile Cinetheodolite Mount with Instrument Installed.

Table II. Cinetheodolite Characteristics

	Low Speed Askania Kth 41	Low Speed Askania Kth 53-58	Hi Speed Askania Land-Air	Contraves
Type Optics	Refractor or Cassengrainian Reflector	Refractor or Cassengrainian Reflector	Refractor or Cassengrainian Reflector	Refractor or Cassengrainian Reflector
Focal Length	60cm-300cm	60cm-300cm	60cm-300cm	60 - 120 in.
F/ratio	4.5 - 8.6	4.5 - 8.6	4.5 - 8.6	8 - 15
Frame Rate	Up to 5/sec	Up to 5/sec	10, 20, 40	10, 20, 30/sec
Film Size	35mm	35mm	35mm	35mm
Frame Size (W x H)	Double Frame 1 1/2 x 1 in	Double Frame 1 1/2 x 1 in	Std 35mm 0.65 x 0.70 in	Std 35mm 0.65 x 0.70 in
Magazine Capacity	170 ft	170 ft	400 ft	500 ft
Coded Timing	Included	Included	Included	Included
Aided Tracking		4 improved Kth 53*		Included
Coded Dials		10 in use		
Effective Recording Time	1 fr/sec - 1280 sec 2 fr/sec - 640 sec 5 fr/sec - 250 sec			
Tracking Rates	Manual -- Approx 15°/sec in azimuth and elevation Electro-Servo Aided -- 30°/sec azimuth -- 15°/sec elevation			

* These instruments, with the exception of the Film Transport Mechanism, are equivalent to the Askania Kth 58.

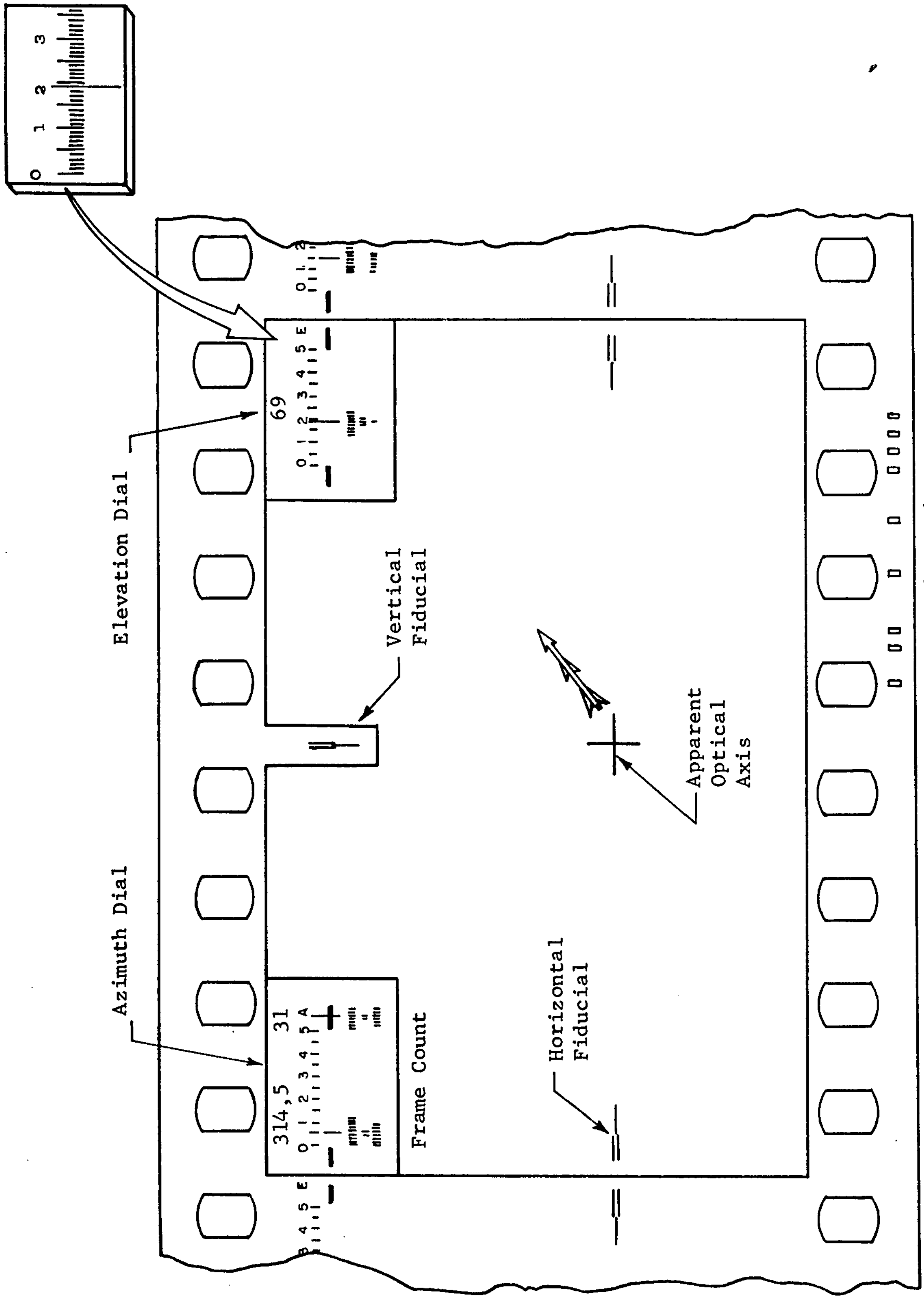

Figure 3. Cinetheodolite film provides several types of data.

2. DESCRIPTION

The cinetheodolite is a combination photo-recording and surveying instrument which tracks and photographs targets (in flight vehicles, etc.) at selectable frame rates. The target image, the azimuth and elevation readings of the cinetheodolite dials, the film frame count, and the binary real time of film exposures are all included on 35 or 70 mm film (Figure 3).

Cinetheodolites may be manual, two man operated instruments; however, some are modified for one man operation, and others utilize power driven azimuth and elevation gearing. Some cinetheodolites have rate-aided tracking control, whereby an open loop servomechanism in conjunction with operator actuated handwheels match the angular rates of the tracking axis with the angular rates of the target line of position. The azimuth and elevation instrument dials provide readings with reference to the instrument optical axis. The displacement of the target from this axis represents the tracking error. Conventionally, using the sighting telescopes and the elevation and azimuth handwheels, one operator adjusts for minimization of the vertical separation of target and telescope reticle images while the other operator adjusts to reduce the horizontal tracking errors.

Cinetheodolites consist of a stable base and bearing; a vertical gimbal or trunnion carrier which rotates about a vertical axis normal to the plane of the base; a central drum or housing which contains the system telescopic lenses, plus a camera and film assembly; a horizontal trunnion shaft on which the central drum is mounted so that it can rise or dip about the horizontal axis; and the sighting telescopes, which are also mounted on the horizontal trunnion shaft.

Different types of shelters and different types of astrodome structures are used to house permanent cinetheodolites. Some shelters house either one or two instruments. In these shelters the cinetheodolites are raised hydraulically to their operating positions. Other shelters feature retractable roofs that open to expose the instrument for operation. In the astrodome structure, the housing is slaved to the cinetheodolite which is mounted on a concrete base. The dome is servo driven for traverse action. A panel of the dome retracts to provide the operators and instrument the necessary field of view.

3. ASKANIA CINETHEODOLITES

Continual improvements in design and long life construction have perpetuated the Askania family of cinetheodolites. Low speed and high speed instruments are available. The low speed Askanias use a pulse operated intermittent film transport mechanism with frame rates of up to five per second. The high speed instrument uses a cine-type mechanism: The film is in constant motion past the driving sprockets and pauses only long enough at the aperture for exposure. The high speed unit can be synchronized to run at rates of 10, 20, or 40 frames per second. Table II gives characteristics of Askania instruments in actual use at a missile range. (A typical model is shown in Figure 4.) This has four functional subsystems; the camera-optical system, the tracking system, the angle-recording system, and the timing synchronizing system.

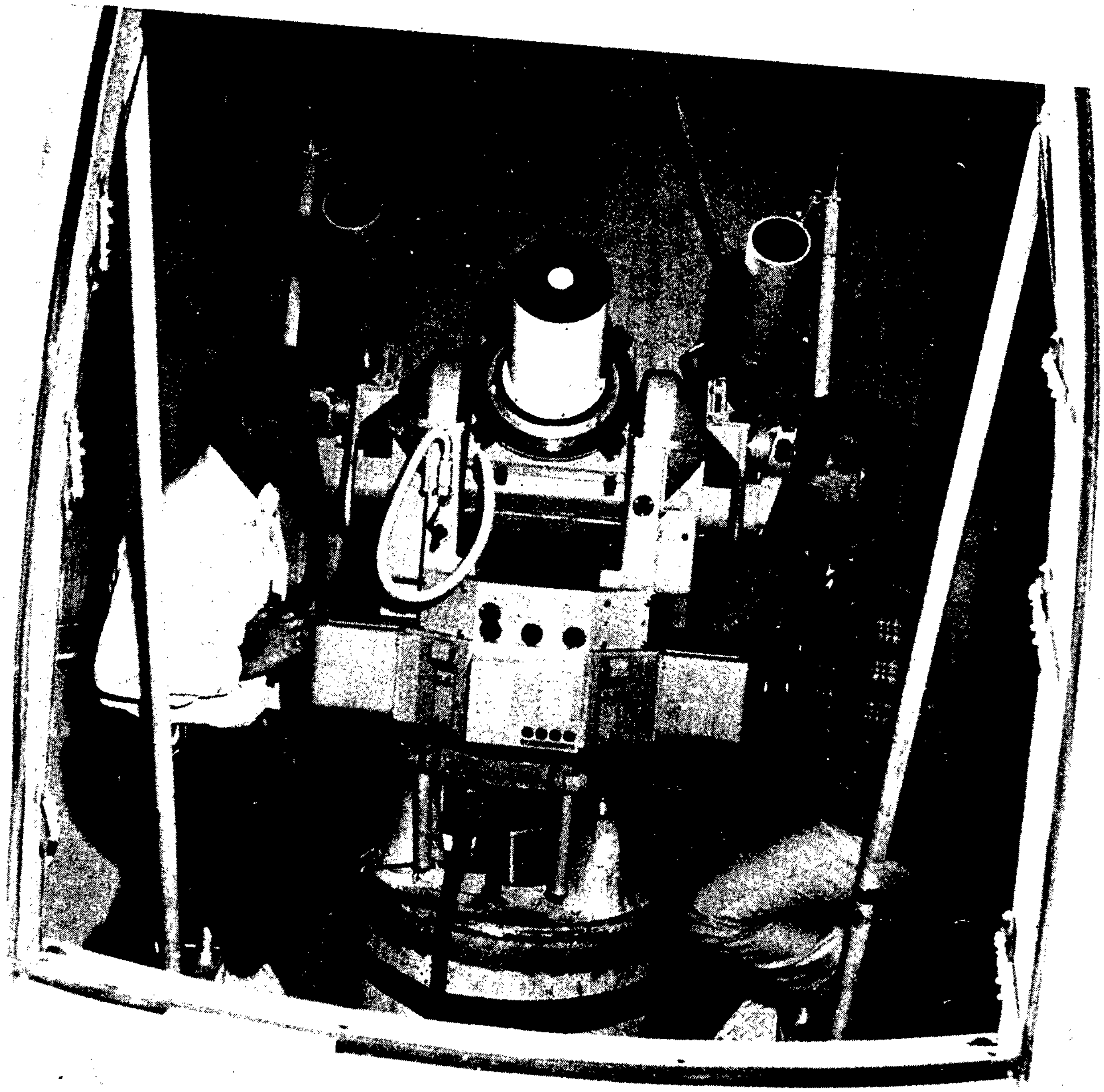

Figure 4. Askania Cinetheodolite.

The camera-optical system includes the objective, shutter mechanism, film gate, film transport mechanism, and frame counters. Objectives are chosen for each specific type of test, and can vary between 30 and 300 cm focal length. The 60 and 120 cm focal length lenses are the most common. The main shutter mechanism consists of overlapping vanes. A maximum pulsed rate of ten exposures per second is permitted at durations of 1/150 to 1/200 second. The film magazine will hold up to 110 feet of 35 mm film.

The tracking system of the Askania design places the camera within the central drum or main barrel which carries the optical system in elevation. The camera is thus maintained fixed in relationship to the telescope. Attached to the trunnion shaft so they remain parallel to the telescope axis are the sighting telescopes. Each sighting telescope contains a reticle whose image the operator attempts to superimpose on the target image by adjusting the elevation and azimuth controls.

The angle-recording system consists of separate optical trains and synchronized high intensity flash lamps which effect photography of graduated glass scales on the upper corners of each film frame. Reticles in the optical trains provide for readings to minutes of arc or to hundredths of a degree. Synchronized flash lamp illumination assures precise time of dial recording, and prevents blurring of the dial images due to tracking motion.

The timing synchronizing system controls shutter timing and dial flash lamp illumination. The synchronizing system is actuated by electrical coded pulses received from a centralized timing control station. The inherent delay in the shutter mechanism is reflected in the advance of the shutter command pulse ahead of the dial flash lamp pulse. Figure 5 shows the sequence of timing events for exposure of one frame of an Askania film record.

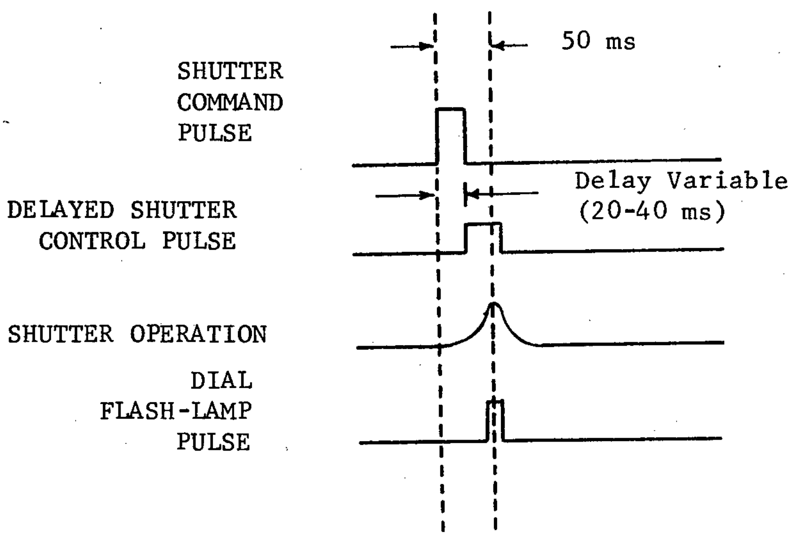

Figure 5. Sequence of Timing Events for Exposure of One Frame of an Askania Film Record.

Simple coding is possible by omitting dial flash-lamp pulses periodically to assist film assessors in correlating or locating frames. Two or more neon bulbs are incorporated at the edges of the frames for exposing serial coding marks on the film while it is being transported.

4. CONTRAVES CINETHEODOLITE

The Contraves cinetheodolites afford improved boost phase coverage and better long range photographic capability. These cinetheodolites (Figure 6) record single frame images on 35 mm film at rates of from 5 to 30 frames per second. Contraves cinetheodolites have been designed for smooth, sided tracking where high angular rates are required. Latest models provide a selection of lens systems with focal lengths of 60 and 120 inches. Table II gives characteristics of Contraves instruments in actual use at a missile range.

The Contraves' film magazine is mounted on the outside of the main housing. This results in the reference system and the target image on the film rotating as a function of instrument elevation angle. Two different film magazines may be used: one holds 400 feet, the other 1000 feet of 35mm film.

At missile ranges, Contraves instruments are used for launch and boost phase coverage, and for intercept coverage close in. In other applications, these instruments have found their largest use in the assessment of aerial fire control systems. More recently, Contraves cinetheodolities have become the major source of accurate data on aircraft trajectory during experimental landings.

The target image may be viewed through the main objective by means of a prism near the camera. The several images of the complete frame to be photographed, including reticles, angle scales, and frame count are all viewed just as the camera will record them. This feature simplifies focusing, and permits rapid field checks.

The main pedestal incorporates seating provisions for the operators. Their sighting telescopes have variable magnification capabilities as a further aid in tracking. Normally tracking is a two man operation; however, modifications may be made to allow for single operator tracking.

As a rule-of-thumb resolving power of the Contraves optical system is felt to be 0.001 inch (0.0254mm). However, for photographic recording purposes the resolution may be no better than 0.005 inch (0.127mm) depending on the photographic materials used.

Master synchronizing pulses fire flashing lamps to illuminate the angle scales and the frame counter such that the camera shutter exposes the target image coincident with flashing lamp illumination.

Position control of the optical axis is by means of a velocity servo. Handwheel displacement varies the rate of tracking. Speed of handwheel displacement is translated into optical axis displacement to give smooth tracking. Predetermined program functions may be introduced to establish the drive servo

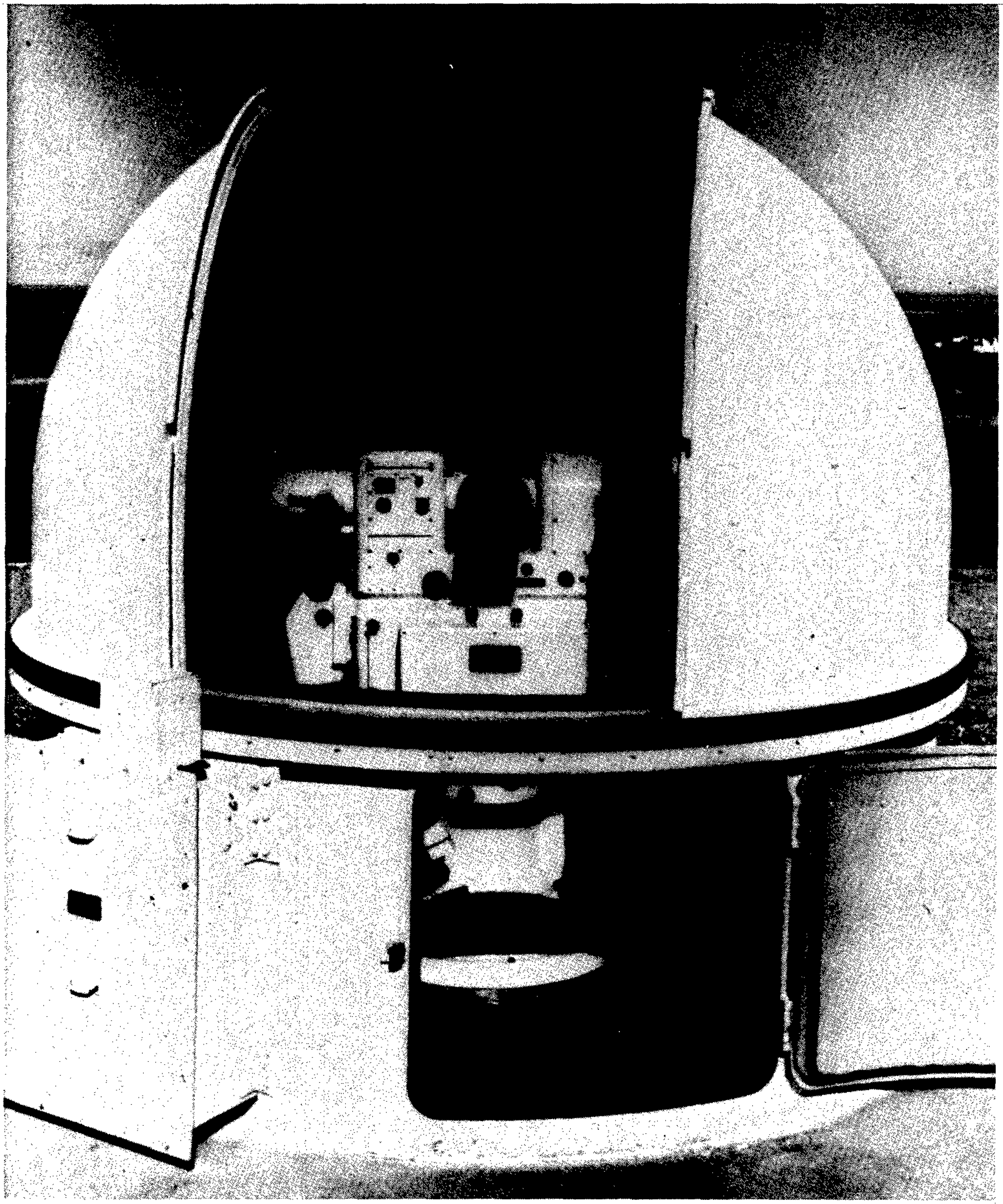

Figure 6. Contraves Cinetheodolite.

slewing rates. Synchros may be added to aid in remote axis positioning from parallax computers operating on data from tracking radars or other equipments in the instrumentation network.

5. DEPLOYMENT OF CINETHEODOLITES

5.1 LIMITATIONS

5.1.1 Base Materials

The actual ground upon which the cinetheodolite is placed should be dry, firm, and level. Locations at elevated sites with towers and domes are preferred for permanent stations and required for full realization of accuracy and detection capability. Slabs of armor plate may be used as bases for semi-permanent or temporary stations.

5.1.2 Tracking Distance

Accurate prediction of the detection capabilities of cinetheodolites requires a very detailed analysis. However, for general information it can be said that the distance at which a cinetheodolite can successfully record an object is a function of three major parameters (Figure 7). The first is the minimum size of image on the film plane that can be tolerated. Normally this is approximately 0.004 inch (0.1016 mm). The second is the focal length, f. The third is the least dimension, i, of the object.

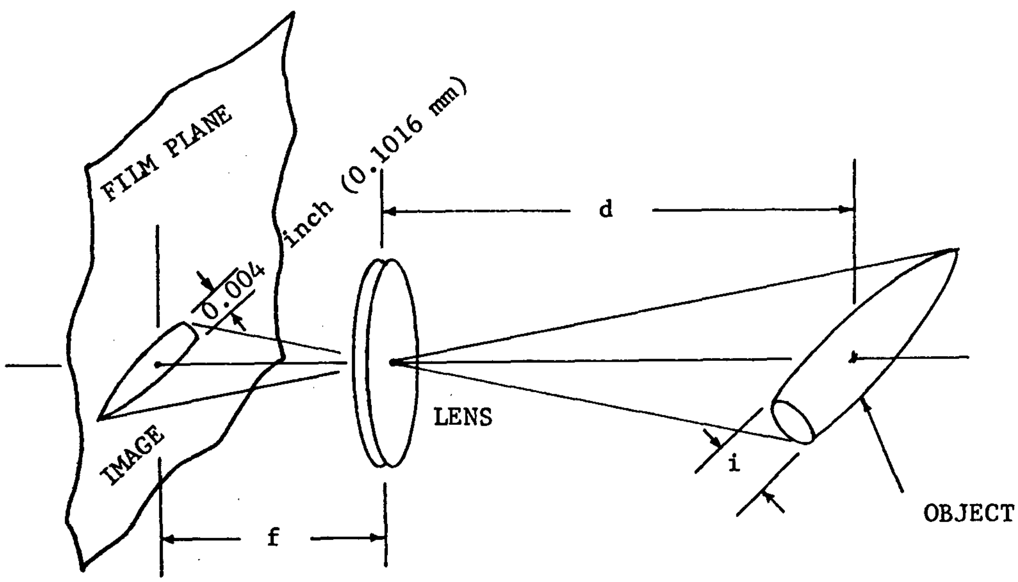

Figure 7. Tracking Distance Factors.

In the oversimplified example of Figure 7, if the least dimension, i, of the object is known, the theoretical maximum distance, d, of that object from the lens can be calculated for any specified focal length. Equation (1) gives the relationships, where all measurements are in meters.

$$d = \frac{if}{1.016 \times 10^{-4}} \tag{1}$$

It should be noted that in practice the maximum distance, d, is frequently lessened due to haze, sun interference, refraction error, etc.

5.1.3 Slewing

Rapid change in angular orientation, as compared to normal tracking angular motion, is termed slewing. The capability of an instrument to accomplish such a maneuver and then permit normal tracking operations constitutes a significant limitation of cinetheodolites. Major factors which affect this instrument capability are the mechanical inertia of the tracking mount, and the angular velocity of the target relative to the instrument (or the distance of the target from the cinetheodolite).

5.1.4 Angular Limitations

Angular limitations result from the use of two or more stations. The baseline distance between stations should be sufficient to provide optimum azimuth intersection angles along the significant portions of the test object's trajectory. Azimuth intersection angles from 60 to 135 degrees and cinetheodolite elevation angles not exceeding 60 degrees provide the best results. As a rule-of-thumb, the least geometrical dilution of precision occurs when the two lines of sight intersect orthogonally at the target, and when ground track of the trajectory bisects the baseline at right angles.

5.2 LAYOUT VARIATIONS

The limitations discussed in paragraph 5.1 above and the data reduction method discussed in 8 below determine the number and locations of the cinetheodolites used for any one mission. The number of instruments or stations and their geometry with respect to the target trajectory are referred to as layout variations.

The simplest layouts are the dual station layouts. In Figure 8 both stations are located to one side of a trajectory. One variation of the layout shown can be made by having the target trajectory bisect the baseline between the stations. For greater coverage, cinetheodolites operating in pairs can be set up in a cascading arrangement down the length of the trajectory.

Multi-station deployment involves the use of more than two cinetheodolites. Since more than one measurement of target position (one measurement per point) is obtained, this allows making a mean position determination which

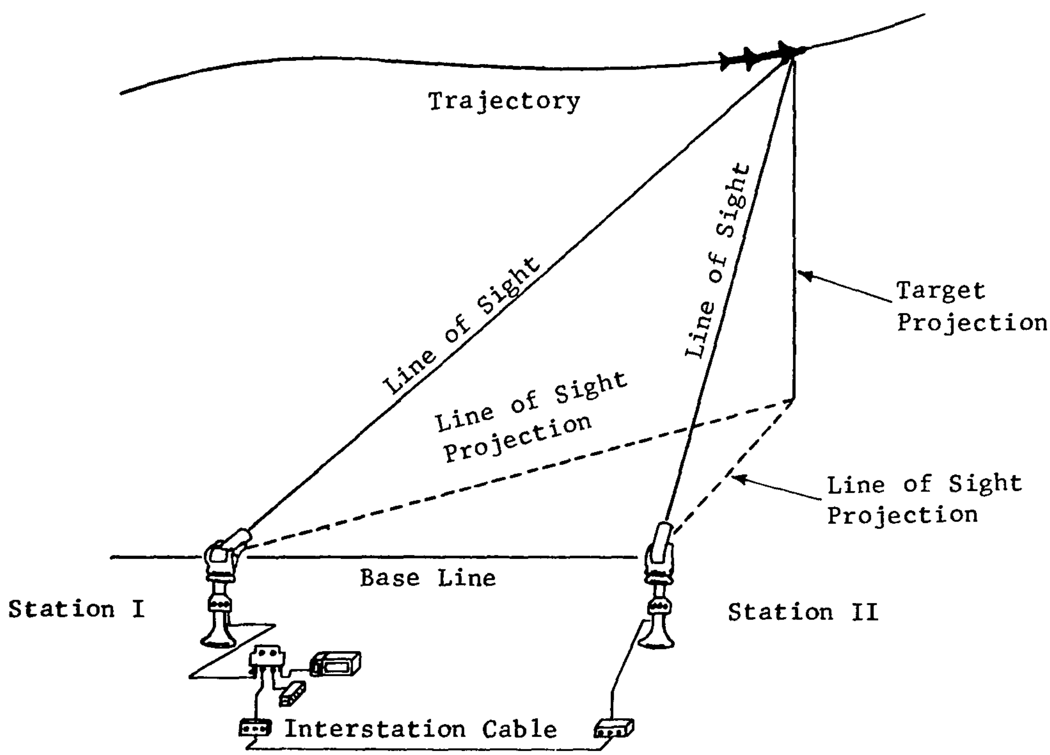

Figure 8. Dual Cinetheodolite Layout.

may be of significant value. It is to be noted that to acquire close to maximum precision four instruments per point are needed. Sometimes redundant data is desired and more than 4 instruments per point are utilized. In multi-station layouts, station location problems may not be much more complicated than those encountered with dual station layouts. In using multi-station systems, proper establishment of the relative locations of each station may result in simplification of data reduction and better precision.

6. SUPPORT EQUIPMENT

In general each cinetheodolite station requires a pulse receiver amplifier, a communications receiver-transmitter, and a 12-volt power supply. In a missile range there may be over 100 instruments; some of them are permanently emplaced, while others are installed in mobile mounts, in temporary, or in semi-permanent stations. Therefore, the support equipment for the operation of each station may be either permanently emplaced as at permanent sites, or mounted on trailers or mobile units which may be readily taken from one place to another as the need arises.

When mobile cinetheodolites are used, additional support equipment is necessary for system operation. Normally a flat-bed truck is used to relocate the trailer-mounted cinetheodolite, and a generator and equipment shelter are installed on the flat-bed.

In the case of multi-station systems, a master control site is required. Generally such a site consists of a frequency standard and coder transmitter, a time generator transmitter, a time recording device, and a communications receiver-transmitter. The hookup of the pulsing equipment for synchronization between stations and master control can be accomplished either by wire or radio.

Target boards are located near each station for precise alignment of axes. Several target boards with or without collimated light sources, establish a reference plane for elimination of pointing errors in the system.

7. CINETHEODOLITE DATA

Cinetheodolites produce records which are not usable until they are developed and processed. Once processed, the cinetheodolite film is observed in a Dial Reader or similar device by trained operators. The dial markings can be read to an accuracy of 6 seconds of arc, and the tracking corrections can be read to an accuracy of 4 seconds of arc. Figure 3 shows a frame of a cinetheodolite record. Table III presents cinetheodolite data acquisition summaries with typical slant range resolutions for boost, mid-course, and terminal phases of test flights at a missile range. Appendix B presents a discussion of data flow and processing at White Sands Missile Range.

Errors may be introduced due to faults in the instruments or in the system. Instrument errors include eccentricity, nonperpendicularity or standards error, lens sag, collimations and bearing wobble. System errors may include mislevel, incorrect film reading, and tracking errors. Prior to and/or during

Table III. Cinetheodolite Data Acquisition Summaries

Phase	Sampling Rate	Data Conversion Equipment	Resolution: Position	Resolution: Velocity	Resolution: Acceleration
Boost See Note 1	Up to 30/sec	Cinetheodolite Dial Reader	15,000 feet slant range ± 2 feet	1 ft/sec	0.5 ft/sec^2
Mid-Course See Note 2	5/sec (60/sec in 50 mile area)	Cinetheodolite Dial Reader	100,000 feet slant range ±15 feet	10 ft/sec	5 ft/sec^2
Terminal See Note 3	5/sec (60/sec in 50	Cinetheodolite Dial Reader	15,000 feet slant range ± 2 feet	1 ft/sec	0.5 ft/sec^2

Note 1. The boost phase applies to ground launch tests after flight stability has been established. Generally data between 500 feet and 20,000 feet are included in this category.

Note 2. The mid-course phase refers to the cruise portion of the test. It includes data on maneuverability, flight stability, miss distance, etc. This category generally refers to data collected between 1000 feet and the altitudes of satellite orbits.

Note 3. The terminal phase refers to all tests which require the collection of data clear into a ground impact. Generally data collected between 15,000 feet and 0 feet are included.

the data acquisition process, tracking aids and calibration procedures are utilized to minimize the effect of errors; while during the data interpretation process, compensation is made for errors to minimize or nullify their adverse effect. Appendix A presents a discussion of accuracy determination at White Sands Missile Range. Paragraph 10 discusses some error minimization procedures.

8. DATA REDUCTION METHOD

There are three methods for determining the spatial position of a target from angular measurements such as those obtained by cinetheodolites. These methods are: intersection triangulation, resection triangulation, and stereo-triangulation; however, intersection triangulation is the most widely used for cinetheodolite instrumentation.

In reducing angular measurements data, the first requirement is accurate knowledge of each cinetheodolite's orientation in terms of a common coordinate system. Then intersection triangulation involves the selection of the most probable point in space at which rays from the target to two or more cameras intersect. If the baseline distance between stations and the base angles of the resulting triangle are known, the position of the target relative to the stations can be determined. An excellent discussion of the geometrical and analytical details involved in these calculations is contained in Reference C of this MTP.

9. PROBLEMS ASSOCIATED WITH CINETHEODOLITES

9.1 SYSTEM AND INSTRUMENT ERRORS

Errors may be introduced because of faults in the instrument or the system. Instrument errors include eccentricity, nonperpendicularity or standards error, lens sag, collimation, and bearing wobble. System errors may include mis-level, scale errors (sometimes termed incorrect film reading), and tracking errors. Figure 9 depicts several of these and other sources of inaccuracies in cinetheodolites. Figure 10 illustrates azimuth error ΔA caused by the combination of collimation error c, and standards error k. If those two errors were non-existent, the true target position along line A - Zenith, and points A and $\underline{A}$ on the horizon would also be coincident, eliminating azimuth error $\underline{\Delta A}$.

Scale errors exist when either the azimuth or elevation scales are imperfect with regard to spacing (graduation error) or with regard to centering (eccentricity). The graduation error may produce random inaccuracies in the order of 2 or 3 seconds of arc; however, inaccuracies due to this fault are more often negligible. Figure 11 illustrates eccentricity of angular scale. This fault produces an error which is approximated by

$$e = \frac{E}{R} \sin A \qquad (2)$$

where e is the error in radians, E is the distance between the center of rotation

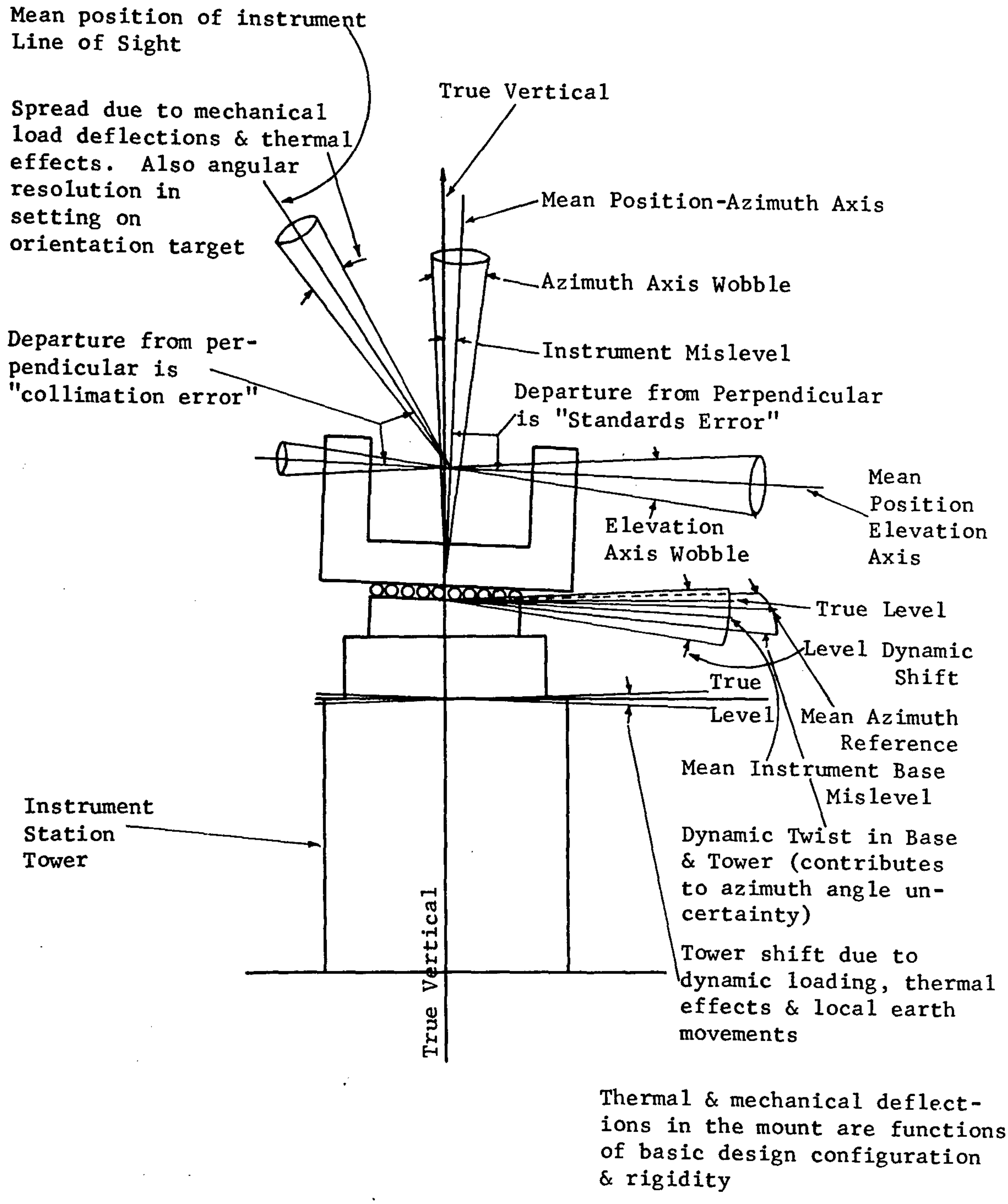

Figure 9. Sources of Inaccuracies in Cinetheodolites.

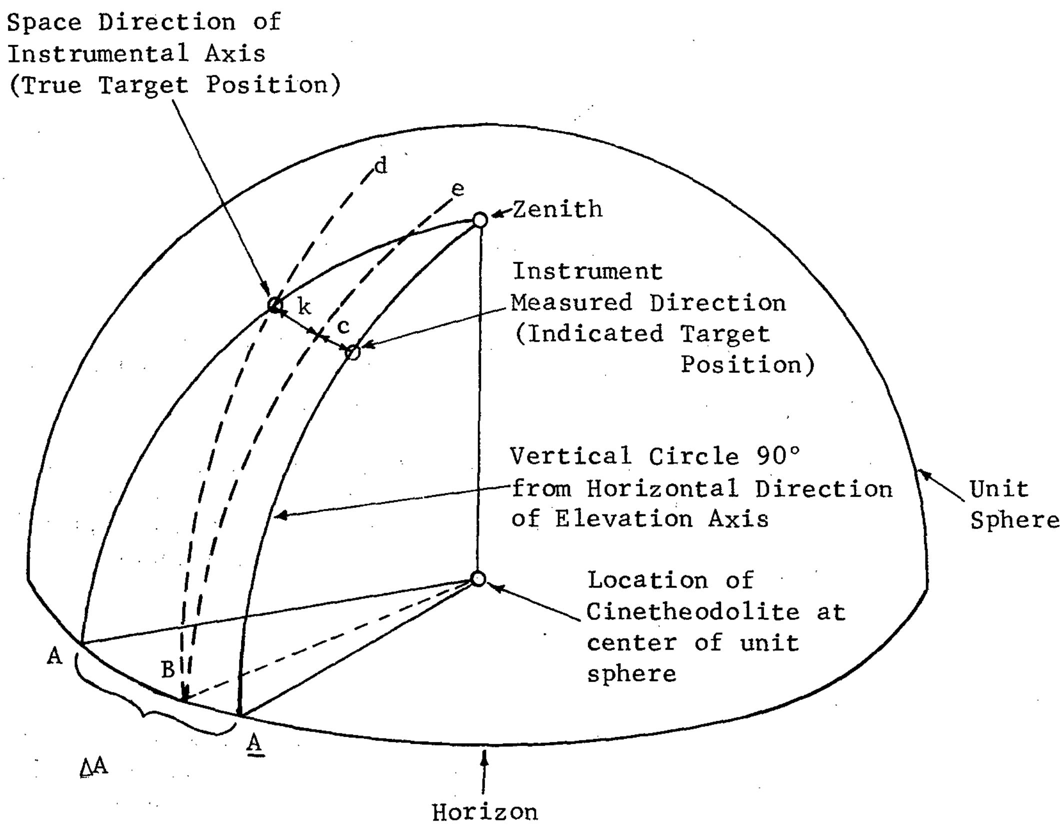

Figure 10. Azimuth Error ΔA.

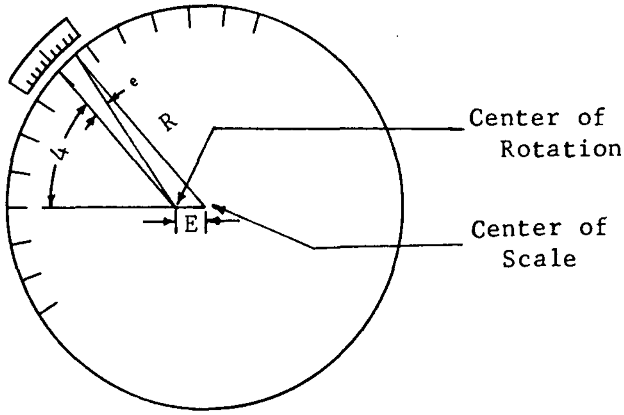

Figure 11. Eccentricity of Angular Scale.

and the center of the scale, R is the radius of the scale, and A is the angle between the direction in which the measurement is being made and the line joining the center of the scale and the axis of rotation.

Elevation scale readings may be further subject to inaccuracies by lens sag. This may be of greater significance when the instrument is fitted with a long focal length optical system and when the elevation approaches 0°, which produces a bending phenomenon comparable with that affecting gun tubes.

9.2 ATMOSPHERIC FACTORS

9.2.1 Visibility

Optical instrumentation may be compromised by fog, haze, smoke, dust, clouds, precipitation, or failure of the target to be distinguished from its background. Sunlight reaches the target from different directions as time of day and target position change. Reflections from the target to the optical instrument change constantly with target aspect and angle of illumination. Brightness of the sky in the background changes with time of day, atmospheric conditions, and the direction of the line of sight. The brightness of certain types of paint may vary with the angle of light incidence. All of these variables contribute to a wide variation of optical visibility.

9.2.2 Contrast

Contrast depends on the brightness of the background and the brightness of the target at zero distance and vice versa. In cinetheodolite work, a condition of zero distance does not exist; therefore, as target distance increases contrast is reduced by the combined effects of atmospheric scattering, atmospheric absorption, contrast transmittance of the atmosphere, and the contrast threshhold of the observer. At near distances, the image of the target reveals details of shape. At some greater distance, the target image is merely the diffraction pattern of a point source. As a rule-of-thumb minimum brightness

of twice the brightness of the background to the target or vice versa is necessary for successful cinetheodolite instrumentation.

Greatly dependent on contrast, the minimum acceptable image size is a matter of local definition. Film grain size, other objects in the background, change in image position from frame to frame, the needs of particular types of measurement, and personal qualifications of the film reader affect the image size required for positive identification. However, a diameter of not less than 0.004 inch is considered generally acceptable by film reading personnel; while an image diameter of less than 0.001 inch is not likely to be satisfactory.

9.2.3 Refraction

The bending of light within an atmospheric path results in refraction errors. When a target and an optical instrument are both within atmospheric strata of substantial density, the light suffers terrestrial refraction. Terrestrial refraction is prevalent when the elevation angle of the line of sight is less than 3 degrees. At higher angles it is sometimes absent; it generally decreases with increasing elevation angle until the effect disappears at elevation angles near 90°. Figure 12 shows the effect of refraction on virtual versus true target positions seen from a single instrument.

9.3 HUMAN FACTORS

With reference to the cinetheodolite operator, apparent contrast depends on the brightness of the horizon, brightness of the target, atmospheric scattering, atmospheric absorption, contrast transmittance of the atmosphere, and the contrast threshold of the observer. This last named factor is of great importance and as a subjective variable, it can constitute a problem in cinetheodolite operation.

Smooth tracking results in target image integration and reinforcement; and conversely, tracking shifts results in a noticeable decrease of point-target image contrast, with consequent impairment of usable data. The ability to track smoothly has been greatly improved by placing the operator on the mount or moving him physically with the instrument so that his walking motion does not affect tracking smoothness.

As missile performance increases, operator reaction-time has become a very important factor, and in the near future most tracking will be done automatically.

10. ERROR MINIMIZATION PROCEDURES

10.1 MECHANICAL DESIGN

Special attention to mechanical design of cinetheodolites has placed all possible loads on an outer column, and only necessary loads of the optical components are placed on an inner column in order to avoid deflection or deformation of bearings. A locking device removes loads from the bearings while the instrument is in transit. Leveling of the instrument is accomplished with coarse

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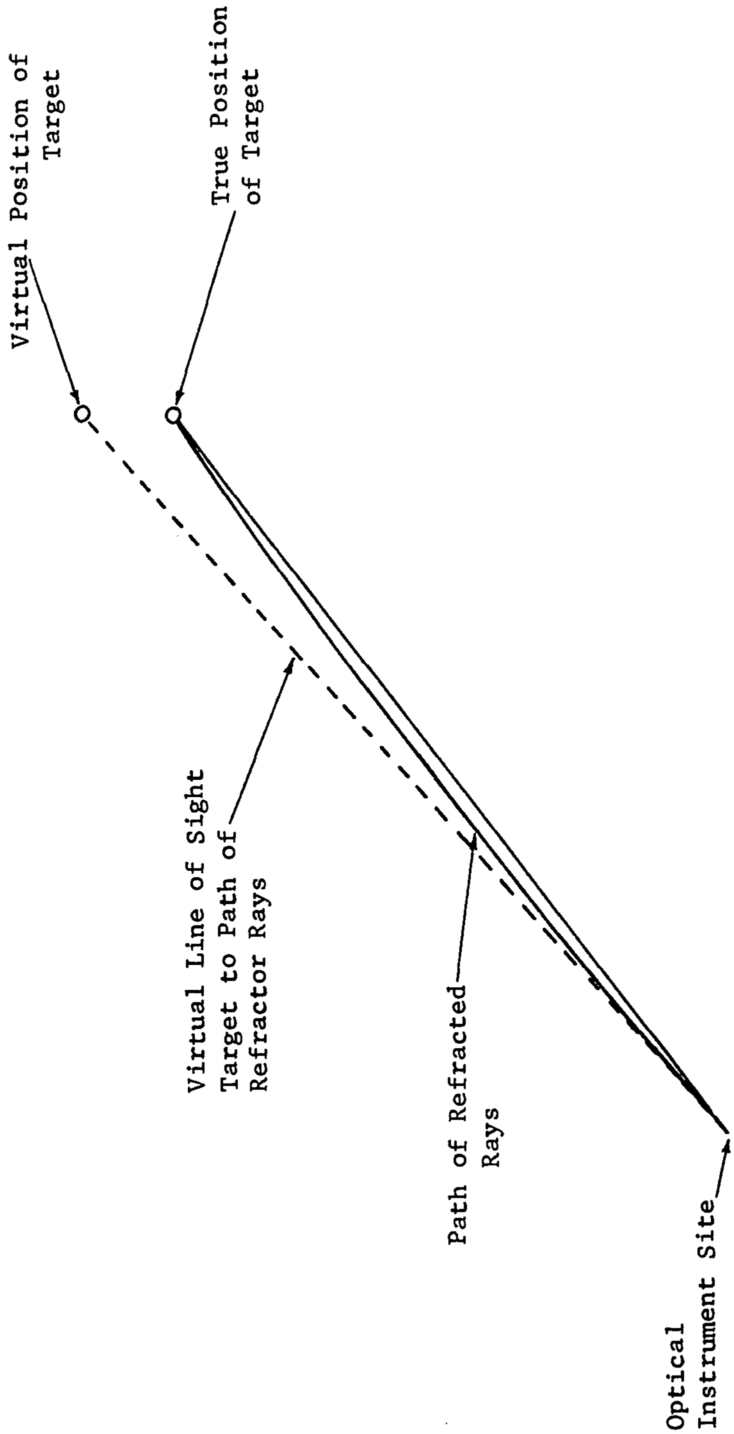

Figure 12. Effect of Refraction on Virtual Versus True Target Positions Seen from a Single Instrument.

and fine screw adjustments to provide a high order of level. Mounting precautions, protection from the weather, and care in use all contribute to preserve the mechanical accuracy of cinetheodolites.

10.2 CALIBRATION

Calibration procedures, as such, are carried out during maintenance or rebuilt of cinetheodolites. However, depending on the time of day of the anticipated flight, the mission, target configuration, and similar factors, premission or postmission procedures are performed for minimization of pointing errors. Essentially these procedures make use of target boards located near each station for precise alignment of axes. Use of several target boards with or without collimated light sources establish a reference plane for each instrument for minimization of pointing errors in the system.

10.3 DATA AND INSTRUMENTATION ERROR ANALYSIS

There is wide variance of opinion concerning data and instrumentation error analysis and/or the methods of accomplishing such analysis. Appendix A presents the manner in which data agencies at White Sands Missile Range accomplish reduction. The examples cited represent typical procedures for cinetheodolites as well as for fixed camera, ballistic camera, and tracking radar.

Essentially, data reduction of cinetheodolite film records from two or more instruments results in space-positions and derivative data such as velocity and acceleration. After adjustment for tilt, orientation, refraction, circle eccentricities, and boresight plus rotation into a common coordinate system, the azimuth and elevation angles from all available instruments for each instant in time are combined to obtain a mathematical estimate of the target position. That point is selected which minimizes the sums of the squares of the angular errors of all angles used in the solution.

REFERENCES

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APPENDIX A
ACCURACY DETERMINATION

1. GENERAL

Table III, paragraph 7, presents cinetheodolite data acquisition summaries with typical slant range resolutions for boost, mid-course, and terminal phases of flight tests. The overall manner in which White Sands Missile Range agencies accomplished the reductions illustrated in above referenced table are outlined in subsequent paragraphs.

2. COMPUTATION PROCEDURES

2.1 UNSMOOTHED POSITIONS WITH ERROR ESTIMATES

Data were computed in the WSCS left-hand rectangular coordinate system (para. 2.2 below) which was translated and rotated to the origin of launcher. The X-axis was positive in the northward direction 0 degrees from grid north; the Y-axis was positive 90 degrees clockwise from the positive X-axis; the Z-axis was positive upward.

Unsmoothed position data were computed using a technique which selects the point such that the sum of the squares of the residual angles, from the observed lines of sight to the point, is a minimum. "N" is the number of stations used in computing the position data at that time.

2.2 WHITE SANDS CARTESIAN SYSTEM

The White Sands Cartesian system (WSCS) is a rectangular system used for reporting instrumentation points and their respective orientation systems.

E is measured, in the plane, along a line passing through the point in question and crossing the North-South axis at a right angle and increasing positively eastward. This value is expressed in feet.

N is measured, in the plane, along a line passing through the point in question and crossing the East-West axis at a right angle and increasing positively northward. This value is expressed in feet.

Z is measured positively upward from, and normal to, the plane.

The origin of this system is the intersection of latitude 33° 05' 00.000" North and longitude of 106° 20' 00.000" West. This origin has a value of E-500,000.00 feet and N-500,000.00 feet. At this point, the plane is tangent to the Clarke Speroid at sea level.

2.3 ERROR ESTIMATES

The true positions, with 90 percent confidence, lie within the published value $\pm a_\sigma$; where σ is the published error estimate, and where

N = number of stations

2N - 3 = number of degrees of freedom

$t_{.10}$ refers to Student's T Distribution

then

$$a = \frac{t_{.10}}{\sqrt{2N - 3}}$$

Number of Stations	a
2	6.413
3	1.359
4	0.901
5	0.718
6	0.611
7	0.541

2.4 SMOOTHED POSITION, VELOCITY, AND ACCELERATION DATA

Smoothed position data were computed by fitting a second degree least squares curve to M consecutive unsmoothed position points having their midpoint at the time of the desired smoothed position.

The smoothed position data and velocity data were computed using an 11-point smoothing interval, or M = 11. The acceleration data were computed using a 21-point smoothing interval, or M' = 21.

Component velocities V(X), V(Y), and V(Z) were computed by taking the first derivative of the second degree least squares curve fitted to M' consecutive unsmoothed position points having their midpoint at the time of the desired acceleration.

2.5 TOTAL VELOCITY, TANGENTIAL ACCELERATION, NORMAL ACCELERATION, AND TRAJECTORY ANGLE DATA

Total velocity data, V, were computed as the magnitude of the vector sum of the component velocities.*

Tangential acceleration data, A(T), were computed by projecting the total acceleration onto the velocity vector.

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* The component velocities used in computing $\dot{\theta}$ and $\dot{\phi}$ are computed from the same smoothing interval used to obtain component accelerations.

Normal acceleration data, A(N), were computed by the vector difference between the sum of the acceleration components and the tangential acceleration.

θ, the elevation angle of the velocity vector, is obtained by:

$$\theta = \frac{\text{arcsine } V(Z)}{V}$$

ϕ, the azimuth angle of the velocity vector, is obtained by:

$$\phi = \frac{\text{arctangent } V(Y)}{V(X)}$$

$\dot{\theta}$, the rate of change of θ, is obtained by:

$$\dot{\theta} = \frac{-V(X)\ V(Z)\ A(X) - V(Y)\ V(Z)\ A(Y) + [V(X)^2 - V(Y)^2]A(Z)}{V^2\sqrt{V(X)^2 + V(Y)^2}}$$

the rate of change of ϕ, is obtained by:

$$\phi = \frac{V(X)\ A(Y) - V(Y)\ A(X)}{V(X)^2 + V(Y)^2}$$

2.6 ERROR ESTIMATES FOR SMOOTHED POSITION, VELOCITY, AND ACCELERATION DATA

The value of $\sigma_{X(S)}$ is computed using a formula of the form

$$\sigma_{X(S)} = \left[\sum_{\frac{-(M-1)}{2}}^{\frac{M-1}{2}} \frac{[X-X(S)]^2}{M-3} \right]^{\frac{1}{2}}$$

where X and X(S) are the respective unsmoothed and smoothed position data for each time, and M is the number of points used in the smoothing interval. Similar equations are used for $\sigma_{Y(S)}$ and $\sigma_{Z(S)}$.

Accuracies of velocity and acceleration data derived from position data are functions of smoothing interval, sampling rate, and position data accuracy. A measure of the accuracy of the mid-point velocity is obtained by deviations from the least squares fit given by:

$$\sigma_{V(X)} = \sqrt{\frac{12\left[\sigma_{X(S)}\right]^2}{M(M^2-1)(\Delta t)^2}}$$

where M is the number of unsmoothed position data points used in the velocity fit and t is the time interval between successive data points.

A measure of the accuracy of the acceleration is obtained by deviations from the least squares fit given by:

$$\sigma_{A_X} = \sqrt{\frac{720\left[\sigma_{X(S)'}\right]^2}{M'[(M')^2-1][M')^2-4][\Delta t]^4}}$$

where M' is the number of unsmoothed position data points used in the acceleration fit, t is the time interval between successive data points, and $\sigma_{X(S)'}$ is the standard error estimate for the position using a M' point smoothing interval.

Where M and M' are equal, $\sigma_{X(S)'}$ is identical to $\sigma_{X(S)}$.

APPENDIX B
DATA FLOW AND PROCESSING

The data acquired through the use of optical equipments such as cine-:heodolites is received and processed at either of two separate White Sands lissile Range facilities.

Raw data are recorded on 35mm film with Askania, Contraves, or other :inetheodolites. The film data are read and recorded on magnetic tape. These ıre then read into the computer for the computation of trajectory data. For ›osition determination, an "n" station least squares Davis solution is used. The 'n" may vary between 2 and 15. In the least squares solution, angular residuals ıre minimized.

The sequence of computations is as follows:

a. Angular calibration constants are determined from target board lata.

b. Raw data are collated from all stations with respect to time.

c. Angular data are corrected for eccentricity, collimation, tracking, tilt and earth's curvature, and refraction.

d. Rotated and translated position data and standard deviations are computed.

Output data per station consist of corrected angles and the following orientation constants: azimuth and elevation angular calibration constants, collimation constants, and I D numbers of the target boards used.

Output data from the "n" station solution consists of rotated and translated position data, standard deviations of position and angles, identification numbers of the station combination per point, and angular residuals in minutes (da_1 and de_1) per station per point.

A summary of the deferred time data processing for cinetheodolites is presented in Table B-I together with the principal parameters of interest and the physical form in which the smoothed data becomes available.

Table B-I. Summary of Cinetheodolite Data and Reduction

Raw Data Form	Type Data	Data Processing Reading Equipment	Output of Reading Equipment
35mm film	Photorecord of a & e angles of camera and missile image with respect to 2 film frame reference points	Teleradex & Askania Dial Reader	Digitized a & e angles and x-y coordinates of missile range position on magnetic tape